$f(x) = \frac{1}{(x-3)(x+2)x} + 1 \quad D = \mathbb{R} \setminus \{0;3;-2\}$

Untersuche $x_0 = 0$

Annäherung von rechts: $x_n = x_0 + \frac{1}{n} \Rightarrow x_n = 0 + \frac{1}{n}$

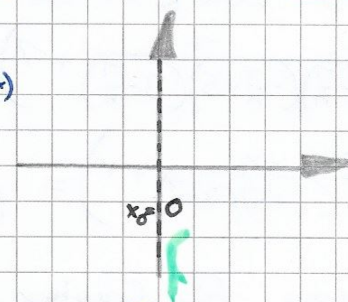
rechtsseitiger Limes:

$$\lim_{n\to\infty}\left(f(x_n)\right) = \lim_{n\to\infty}\left(\frac{1}{(x_n-3)(x_n+2)x_n}+1\right) = \lim_{n\to\infty}\left(\frac{1}{(0+\frac{1}{n}-3)(0+\frac{1}{n}+2)(0+\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{(\frac{1}{n}-3)(\frac{1}{n}+2)\frac{1}{n}}+1\right) = \lim_{n\to\infty}\left(\frac{n}{n}\cdot\frac{1}{(\frac{1}{n}-3)(\frac{1}{n}+2)\frac{1}{n}}+1\right)$$

$$= \lim\left(\frac{n}{(\frac{1}{n}-3)(\frac{1}{n}+2)}+1\right) = \frac{\infty}{(0-3)(0+2)}+1 = \frac{\infty}{-6}+1 = \boxed{-\infty}$$

Bedeutung für Graphen von $f(x)$



Annäherung von links: $x_n = x_0 - \frac{1}{n} \Rightarrow x_n = 0 - \frac{1}{n}$

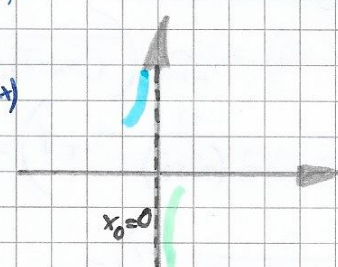
linksseitiger Limes:

$$\lim_{n\to\infty}\left(f(x_n)\right) = \lim_{n\to\infty}\left(\frac{1}{(x_n-3)(x_n+2)x_n}+1\right) = \lim_{n\to\infty}\left(\frac{1}{(0-\frac{1}{n}-3)(0-\frac{1}{n}+2)(0-\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{(-\frac{1}{n}-3)(-\frac{1}{n}+2)(-\frac{1}{n})}+1\right) = \lim_{n\to\infty}\left(\frac{n}{n}\cdot\frac{1}{(-\frac{1}{n}-3)(-\frac{1}{n}+2)(-\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{-n}{(-\frac{1}{n}-3)(-\frac{1}{n}+2)}+1\right) = \frac{-\infty}{(0-3)(0+2)}+1 = \frac{-\infty}{-6}+1 = \boxed{\infty}$$

Bedeutung für Graphen von $f(x)$



Untersuche $x_0 = 3$

Annäherung von rechts: $x_n = x_0 + \frac{1}{n} \Rightarrow x_n = 3 + \frac{1}{n}$

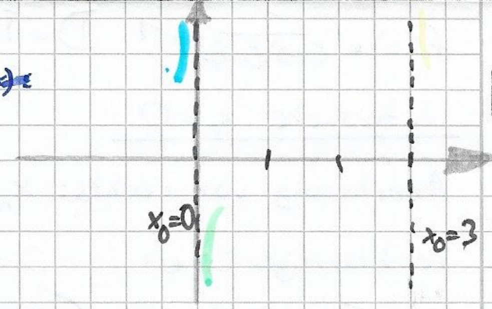
rechtsseitiger Limes:

$$\lim_{n\to\infty}\left(f(x_n)\right) = \lim_{n\to\infty}\left(\frac{1}{(x_n-3)(x_n+2)x_n}+1\right) = \lim_{n\to\infty}\left(\frac{1}{(3+\frac{1}{n}-3)(3+\frac{1}{n}+2)(3+\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{\frac{1}{n}\cdot(\frac{1}{n}+5)(3+\frac{1}{n})}+1\right) = \lim_{n\to\infty}\left(\frac{n}{n}\cdot\frac{1}{\frac{1}{n}\cdot(\frac{1}{n}+5)(3+\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{n}{(\frac{1}{n}+5)(3+\frac{1}{n})}+1\right) = \frac{\infty}{(0+5)(3+0)}+1 = \frac{\infty}{15}+1 = \boxed{\infty}$$

Bedeutung für Graphen von f(x):



Annäherung von links: $x_n = x_0 - \frac{1}{n} \Rightarrow 3 - \frac{1}{n}$

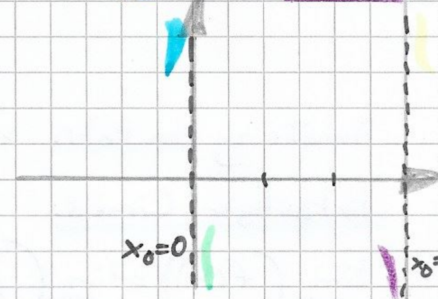
linksseitiger Limes:

$$\lim_{n\to\infty}\left(f(x_n)\right) = \lim_{n\to\infty}\left(\frac{1}{(x_n-3)(x_n+2)x_n}+1\right) = \lim_{n\to\infty}\left(\frac{1}{(3-\frac{1}{n}-3)(3-\frac{1}{n}+2)(3-\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{(-\frac{1}{n})(-\frac{1}{n}+5)(3-\frac{1}{n})}+1\right) = \lim_{n\to\infty}\left(\frac{n}{n}\cdot\frac{1}{(-\frac{1}{n})(-\frac{1}{n}+5)(3-\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{-n}{(-\frac{1}{n}+5)(3-\frac{1}{n})}+1\right) = \frac{-\infty}{(0+5)(3-0)}+1 = \frac{-\infty}{15}+1 = \boxed{-\infty}$$

Bedeutung für Graphen von f(x):



Untersuche $x_0 = -2$

Annäherung von rechts: $x_n = x_0 + \frac{1}{n} \Rightarrow -2 + \frac{1}{n}$

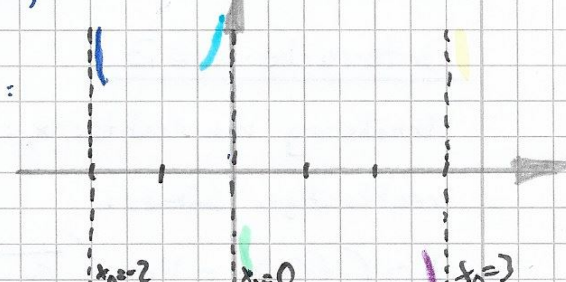
rechtsseitiger Limes:

$$\lim_{n\to\infty}\left(f(x_n)\right) = \lim_{n\to\infty}\left(\frac{1}{(x_n-3)(x_n+2)x_n}+1\right) = \lim_{n\to\infty}\left(\frac{1}{(-2+\frac{1}{n}-3)(-2+\frac{1}{n}+2)(-2+\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{(-5+\frac{1}{n})\frac{1}{n}(-2+\frac{1}{n})}+1\right) = \lim_{n\to\infty}\left(\frac{n}{n}\cdot\frac{1}{(-5+\frac{1}{n})\frac{1}{n}(-2+\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{n}{(-5+\frac{1}{n})(-2+\frac{1}{n})}+1\right) = \frac{\infty}{(-5+0)(-2+0)}+1 = \frac{\infty}{10}+1 = \boxed{\infty}$$

Bedeutung für Graphen von f(x):



Annäherung von links: $x_n = x_0 - \frac{1}{n} \Rightarrow -2 - \frac{1}{n}$

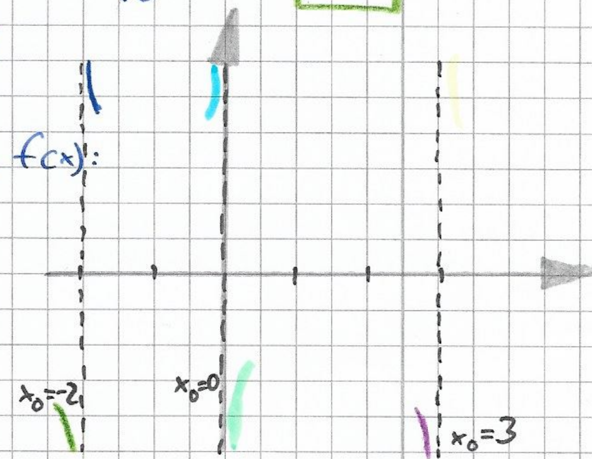
linksseitiger Limes:

$$\lim_{n\to\infty}\left(f(x_n)\right) = \lim_{n\to\infty}\left(\frac{1}{(x_n-3)(x_n+2)x_n}+1\right) = \lim_{n\to\infty}\left(\frac{1}{(-2-\frac{1}{n}-3)(-2-\frac{1}{n}+2)(-2-\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{(-\frac{1}{n}-5)(-\frac{1}{n})(-2-\frac{1}{n})}+1\right) = \lim_{n\to\infty}\left(\frac{n}{n}\cdot\frac{1}{(-\frac{1}{n}-5)(-\frac{1}{n})(-2-\frac{1}{n})}+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{-n}{(-\frac{1}{n}-5)(-2-\frac{1}{n})}+1\right) = \frac{-\infty}{(0-5)(-2-0)}+1 = \frac{-\infty}{10}+1 = \boxed{-\infty}$$

Bedeutung für den Graphen von f(x):



Es existieren drei Polstellen:
$x_0 = \{-2; 0; 3\}$, alle drei sind mit VZW.

Im Vergleich zu den Graphenpuzzle Graph h) sieht man, dass graph h) der passende Graph ist.